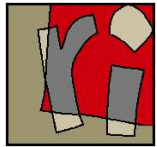


Réseaux Quantiques

éléments de sécurité et de fiabilité de l'informatique
et des algorithmes dans les réseaux quantiques

<http://www.lri.fr/~kempe/ResQuant/>



Julia Kempe

Christoph Dürr

Sophie Laplante

Frédéric Magniez

Miklos Santha

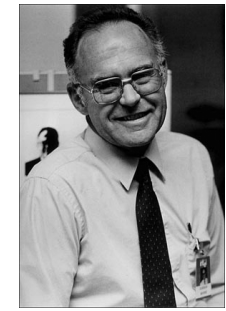


Harold Ollivier

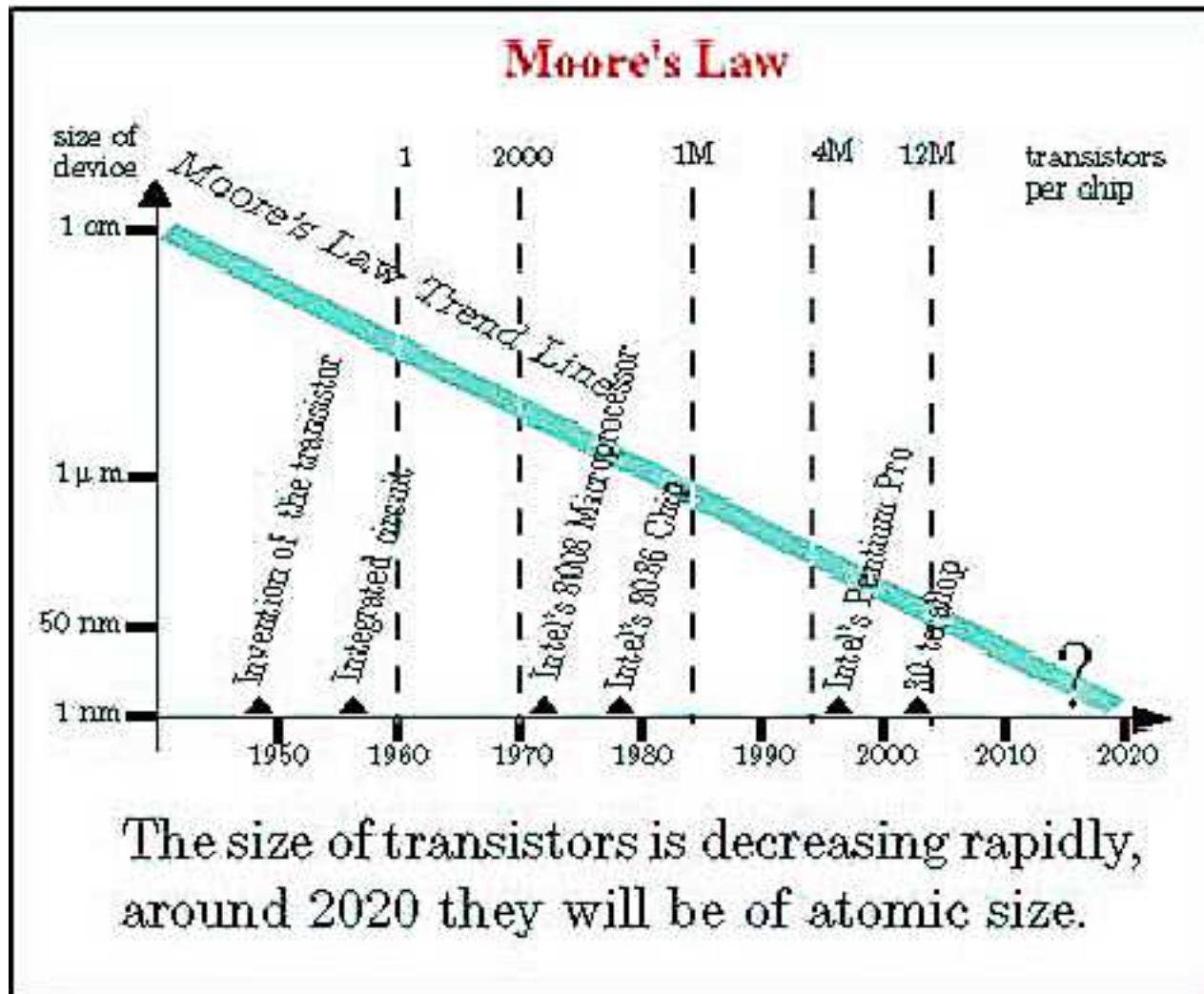
Jean-Pierre Tillich

Jérémy Roland

Fin de la loi de Moore ?



"No exponential is forever. Your job is to forever.", Andrew Gordon Moore Feb. 2011



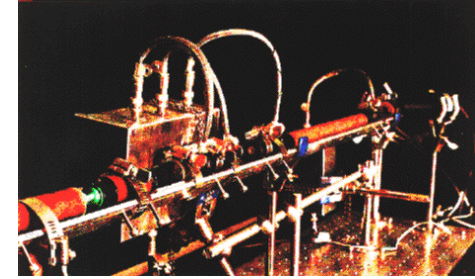
Phénomènes quantiques vers 2020...

- Approche actuelle : les supprimer
- **Informatique quantique** : les utiliser !

Cryptographie

- Protocole de distribution de clés secrètes [Bennett-Brassard 1984]

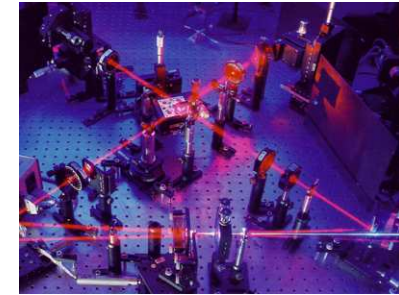
Implémentation : ~100 km



Information quantique

- Téléportation [Bennett-Brassard-Crépeau-Jozsa-Peres-Wootters 1993]

Réalisation : 1997 [Innsbruck]



Algorithmique

- Calcul de périodes [Simon, Shor 1994] Factorisation, log. discret...

- Recherche dans une liste non structurée [Grover 1996]

Implémentation sur combien de qubits ?

1995 : 2 [ENS], 1998 : 3,

2000 : 5 [IBM] - 7 [Los Alamos]

2001 : 8 [IBM]



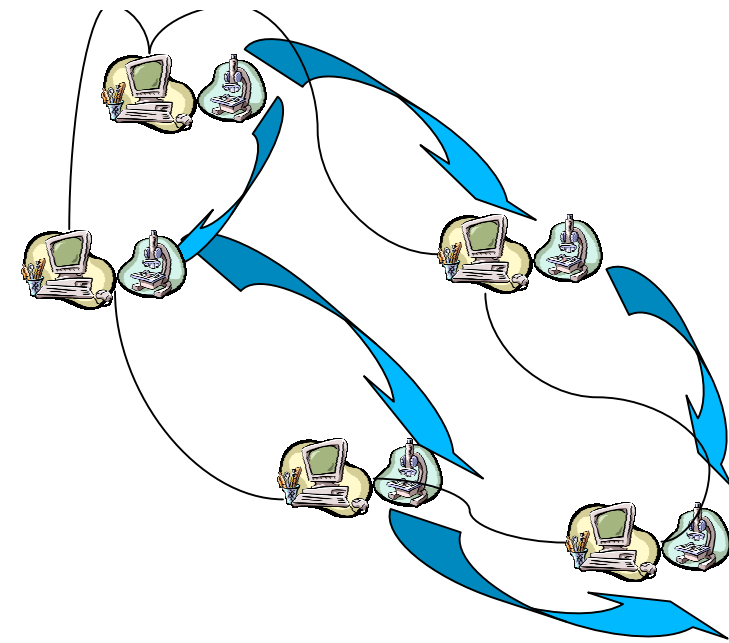
Nature du réseau

-Postes locaux:

Petit ordinateur quantique

Ordinateur *classique*

-Transmission d'information quantique/classique



Objectifs du projet

-Fiabilisation

codes correcteurs quantiques

auto-test des architectures quantiques

-Routage

marches aléatoires quantiques

- algorithmique de graphe

-Nouveaux algorithmes et protocoles de communication

-Characterize le pouvoir des machines quantiques

Quantum convolutional codes, fault-tolerance, and quantum error correction

Ollivier and J.-P. Tillich, "Description of a quantum convolutional code", **Phys. Rev. Lett.** 91 (17), 177901 (2003)

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Shiromiya, H. Ollivier and J.-P. Tillich: "Constructions and performance of classes of quantum LDPC codes", arXiv, quant-ph 0502086 (2005), sousmis

Ollivier and J.-P. Tillich "Interleaved serial concatenation of quantum convolutional codes: gate implementation and error estimation algorithm", in Proc. of 26th **Symp. on Information Theory** in Benelux, 149 (2005)

Shiromiya, J. Kempe, S. Simic, S. Sastry: "Fault-tolerant quantum computation - a dynamical systems approach", **IEEE TAC** (2005)

Ollivier: "Approaches to Quantum Error Correction", bookchapter, **Decoherence**, Progress in Mathematics, Birkhäuser, to appear (2006)

Shiromiya, D. Mayer, M. Mosca, and H. Ollivier, "Self-testing of quantum circuits", sousmis (2005)

Quantum walks/algorithmes sur les graphes

Ollivier: "Discrete Quantum Walks Hit Exponentially Faster", Proceedings of 7th International Workshop on Randomization and Approximation Techniques in Computer Science (**RANDOM'03**), p. 354-69 (2003)

Ollivier, Santha and Mario Szegedy, "Quantum and classical query complexities of local search are polynomially related", **Proc. 36th Symp. on Theory of Computation (STOC)**, p. 494-501 (2004)

Ollivier, M. Heiligman, P. Høyer and M. Mhalla: "Quantum query complexity of some graph problems", **Proc. 35th Symp. on Foundations of Computer Science (FOCS)**, p. 481-493, (2004)

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Shiromiya, M. Santha, and M. Szegedy, "Quantum algorithms for the triangle problem", **Proc. 16th ACM-SIAM SODA**, p. 1011-1020 (2005)

Communication Algorithms

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Kempe and A. Shalev: "The hidden subgroup problem and permutation group theory", *Proc. 16th ACM STOC*, 1118-1125 (2005)

Childs, J. Kempe, S. Myrgren, K.B. Whaley "An explicit universal gate-set for exchange-only quantum computation", *Quantum Information Processing*, Vol. 2 (4), p. 289-307 (2003)

Porocz, J. Vala, K. Brown, J. Kempe, F.K. Wilhelm, K.B. Whaley: "Full protection of superconducting qubits from coupling errors", *Phys. Rev. B*, Vol. 72, 064511 (2005)

Bravyi, J. Kempe and R. de Wolf: "Quantum Communication Cannot Simulate a Public Coin", *lanl-report quant-ph/0511013*, sousmis (2005)

Bravyi, J. Kempe, O. Regev and R. de Wolf: "Bounded-Error Quantum State Identification and Exponential Lower Bounds in Communication Complexity", *lanl-report quant-ph/0511013*, sousmis (2005)

voir des machines/réseaux quantiques

Bravyi and F. Magniez "Lower bounds for randomized and quantum query complexity using Kolmogorov complexity", *Complexity* 2004, p. 294-304 (2004)

Bravyi, W. van Dam, J. Kempe, Z. Landau, S. Lloyd, O. Regev: "Adiabatic Quantum Computation is Equivalent to Quantum Computation", *Proc. 45th FOCS*, p. 42-51, (2004)

Kempe, A. Kitaev and O. Regev, "The Complexity of the Local Hamiltonian Problem", *Proc. 17th STOC*, p. 372-383 (2004)

Olivier, D. Poulin and W. H. Zurek, "Objective properties from subjective quantum states: environment as witness", *Phys. Rev. Lett.*, Vol. 93, 220401 (2004)

Poulin, R. Blume-Kohout, R. Laflamme and H. Ollivier, "Exponential speed-up with a single bit of information: measuring the average fidelity decay", *Phys. Rev. Lett.*, Vol. 92(17), 177906 (2004)

Olivier, D. Poulin and W. H. Zurek "Environment as witness: selective proliferation of information on

Contexte

- Milieu : air libre ou fibre optique
- Transmission : information quantique entre processeurs et mémoires

Solution : codes convolutifs

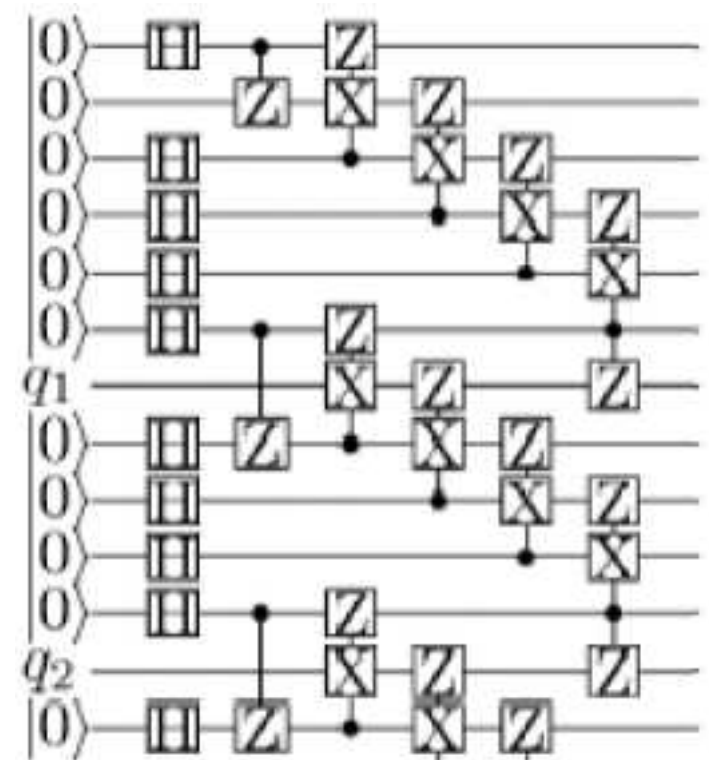
- Codage : complexité linéaire
- Décodage : complexité linéaire
- Application : réduction d'erreur

Exemple

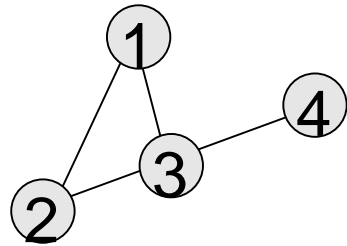
- Code (5,1,2)

Extension

- Turbo-codes ✓
- Décodage itératif ✓
- Codes à matrice de parité creuse ✓



Modèle



Matrice d'adjacence Tableau d'adjacence

	1:	2:	3:	4:
1:	0	1	1	0
2:	1	0	1	0
3:	1	1	0	1
4:	0	0	1	0

1:	2	3	
2:	3	1	
3:	1	4	2
4:	3		

n arêtes m nœuds

Matrice d'adj. Tableau d'adj.

$$\Theta(n^{1.5}) \quad \Theta(\sqrt{nr})$$

$$O(n^{1.3}) \quad O(\sqrt{nr})$$

$$O(n^{2-c})$$

Problèmes

- Connectivité, arbres de plus courts chemins
- Présence de triangle
- Propriété monotone

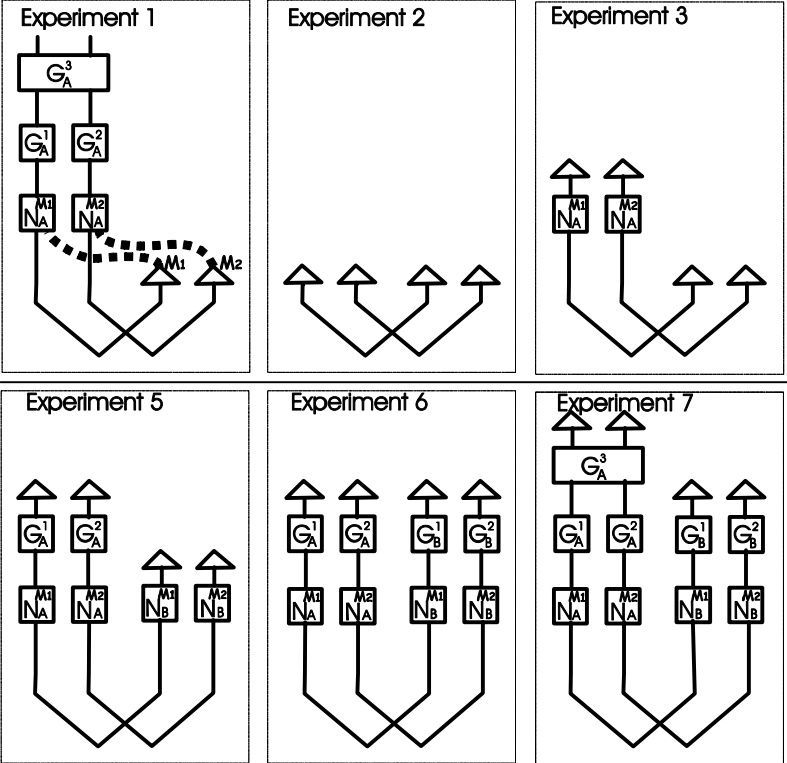
s résultats:

cte

deboguer un ordinateur quantique ?
faire confiance à des fournisseurs de services
s (cryptographiques)?

on : auto-test

une architecture qui puisse être auto-testée
Auto-test de familles de portes
Auto-test inconditionnel d'une paire EPR
ainsi que des appareils de mesure
est robuste!



ésultats:

Contexte

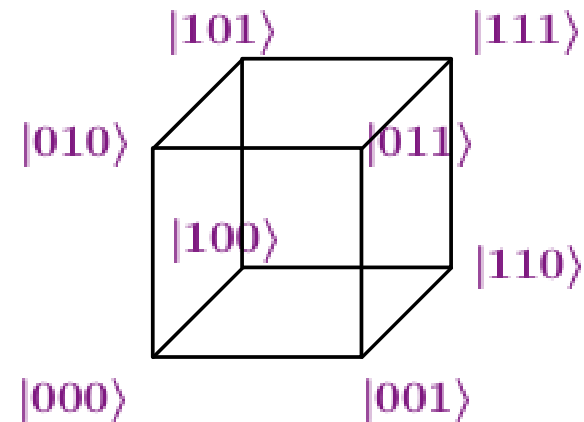
- Information codée sur des photons
- Stockage difficile
- Pas le temps de calculer le chemin !

Solution : marche quantique

- Destinataire aléatoire
- Temps d'atteinte ?

Cas de l'hypercube

- Classiquement : $O(2^n)$
- Quantiquement : $O(n)$!



s résultats:

1: use random walks for network tasks (routing, randomized algorithms)

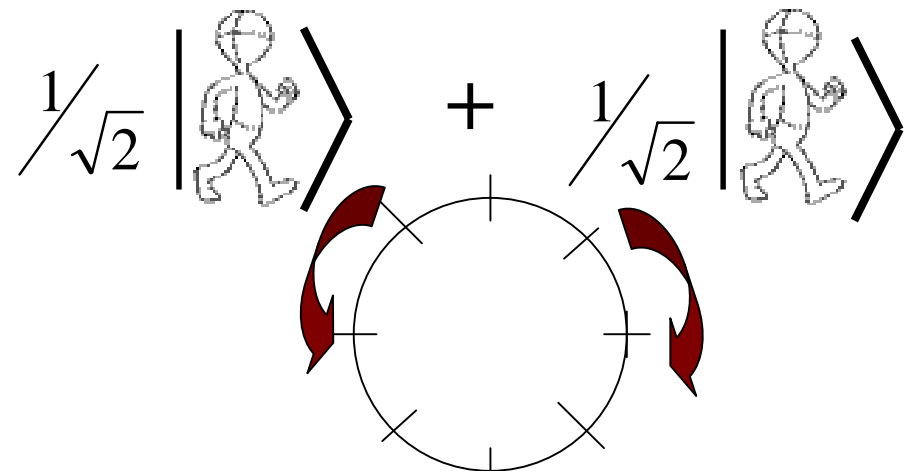
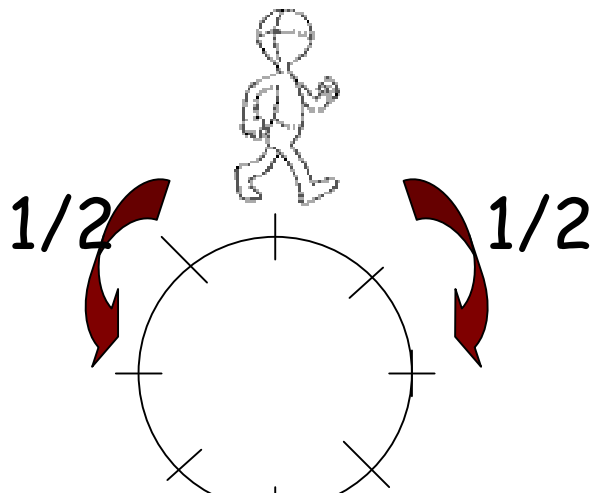
classical: Randomized algorithms very successful

1: construct a Markov chain (**simple, local transitions**)

(1) stationary distribution gives solution to problem \Rightarrow **Mixing time**

(2) which hits the desired solution \Rightarrow **Hitting time**

new tool: Quantum Walks!



Classical random walk

Flip coin for direction
Walk conditioned on
outcome



...	-2	-1	0	1	2
-----	----	----	---	---	---

Quantum "coined" walk:

$$\{|\rightarrow\rangle, |\leftarrow\rangle\} \otimes \dots \begin{array}{|c|c|c|c|c|} \hline \dots & -2 & -1 & 0 & 1 & 2 \\ \hline \end{array}$$

States $|\rightarrow, x\rangle$ and $|\leftarrow, x\rangle$

• Coin flip:

$$|\rightarrow\rangle \Rightarrow \frac{1}{\sqrt{2}}|\rightarrow\rangle + \frac{1}{\sqrt{2}}|\leftarrow\rangle$$

$$|\leftarrow\rangle \Rightarrow \frac{1}{\sqrt{2}}|\rightarrow\rangle - \frac{1}{\sqrt{2}}|\leftarrow\rangle$$

• Shift:

$$|\rightarrow, x\rangle \Rightarrow |\rightarrow, x+1\rangle$$

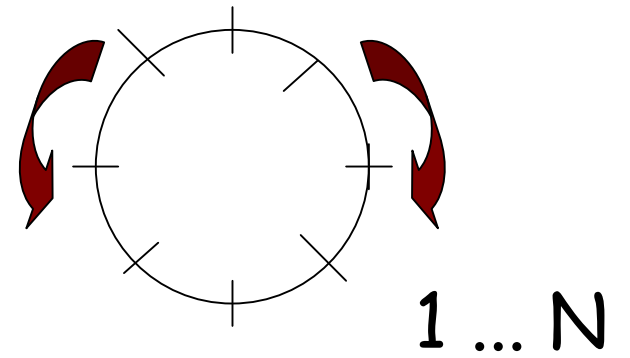
$$|\leftarrow, x\rangle \Rightarrow |\leftarrow, x-1\rangle$$

sample line:

...	-2	-1	0	1	2	...
-----	----	----	---	---	---	-----

g time on N-circle:

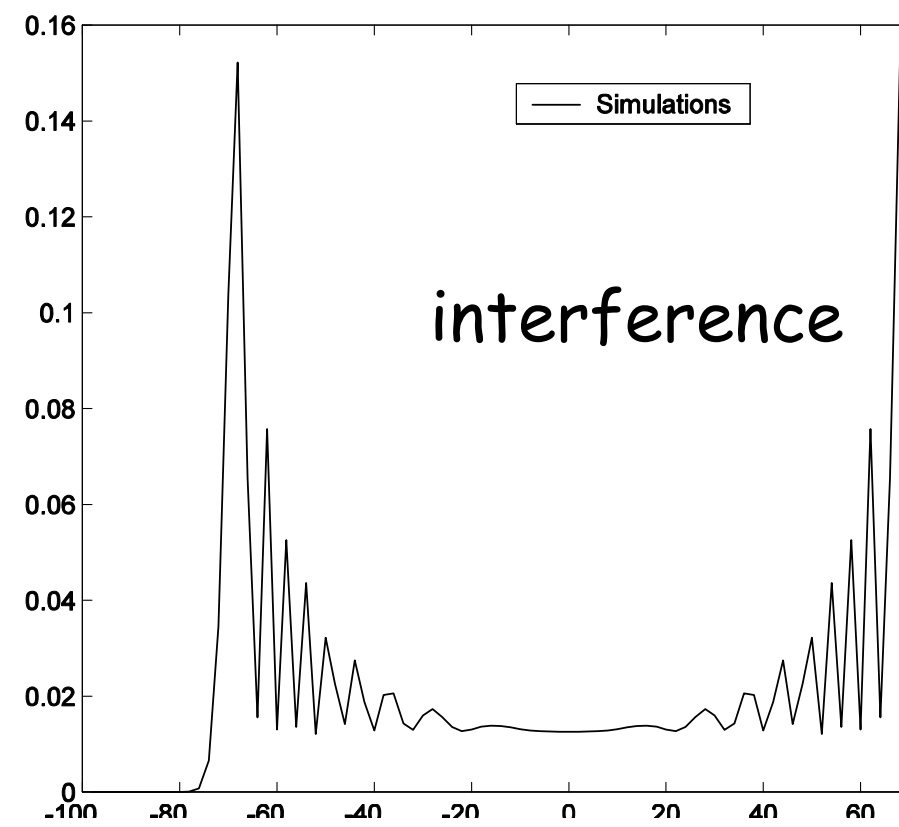
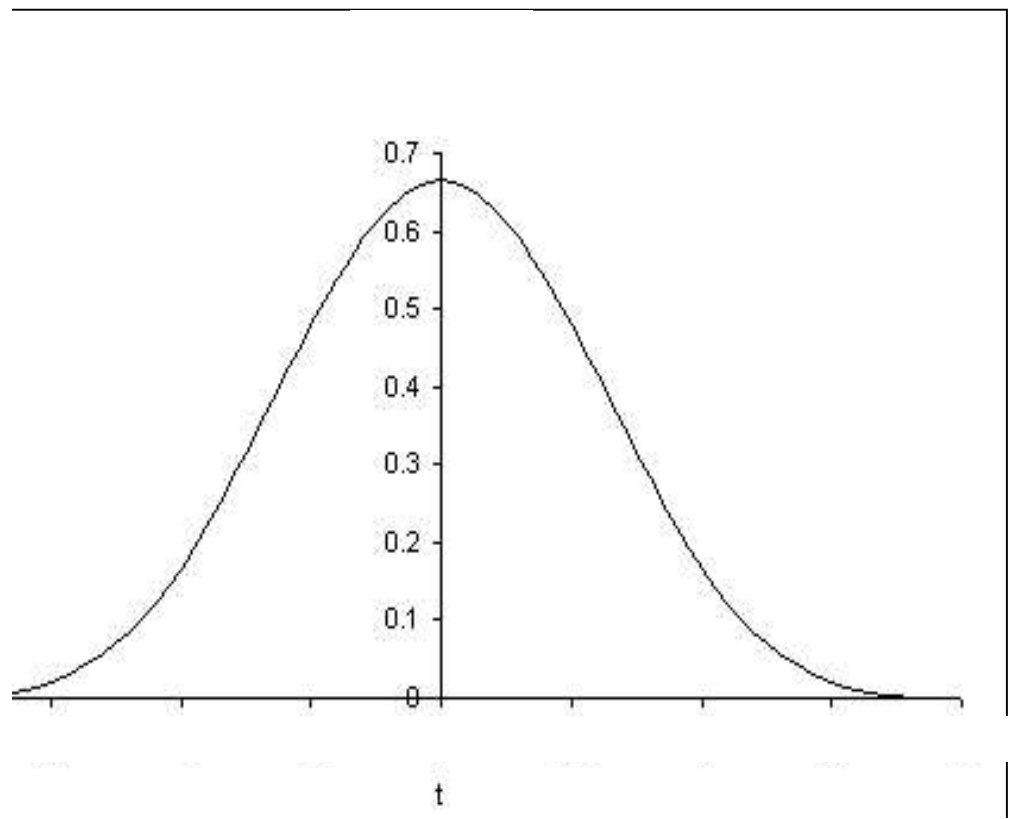
classical: $O(N^2)$ quantum: $O(N)$



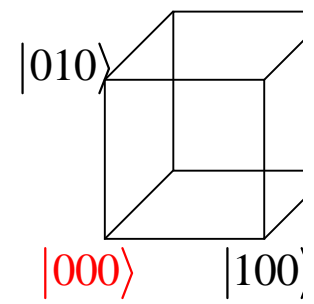
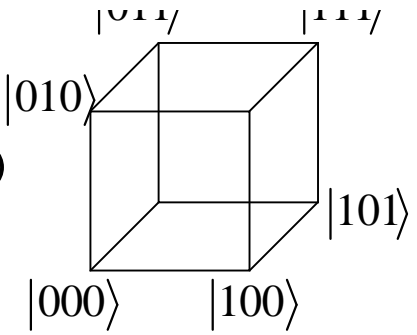
Probability distribution (measurement after T steps):

classical:

Quantum:



Space: $\{|\leftrightarrow\rangle, |\leftarrow\rangle, |\updownarrow\rangle\} \otimes$



Classical: from v to opposite corner, hitting-time exponential

$$T = 2^n (1 + O(\dots))$$

Quantum: polynomial

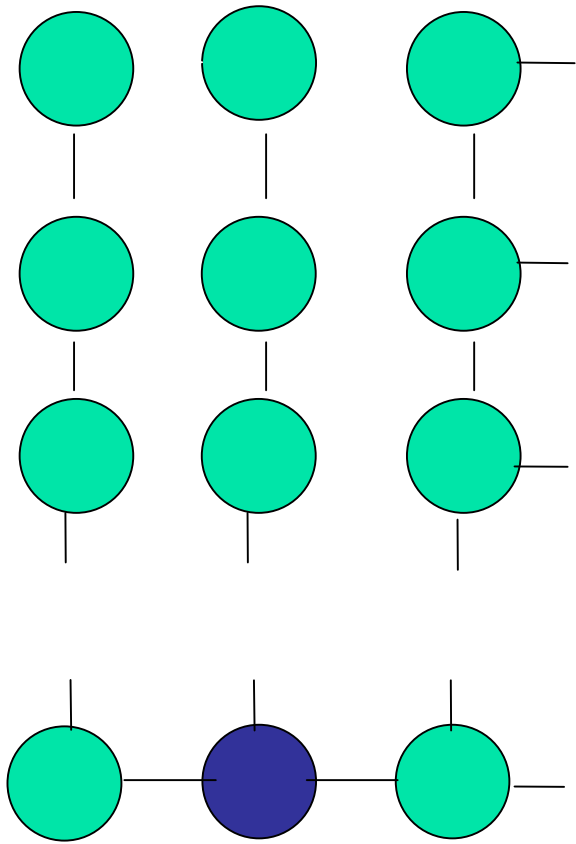
Hitting-time from v to opposite corner :

n known: $T \approx \frac{\pi}{2} n$

n unknown: $T = O(n^2)$

or decentralized routing in networks!
 routing nodes do not know destination of packet, still routing is
 exponential advantage over the classical setting.

Set of items S , some marked.



Algorithm:

Start with a uniform superposition over all S .

Apply one coin (transition rule) if S marked,
another if S not marked.

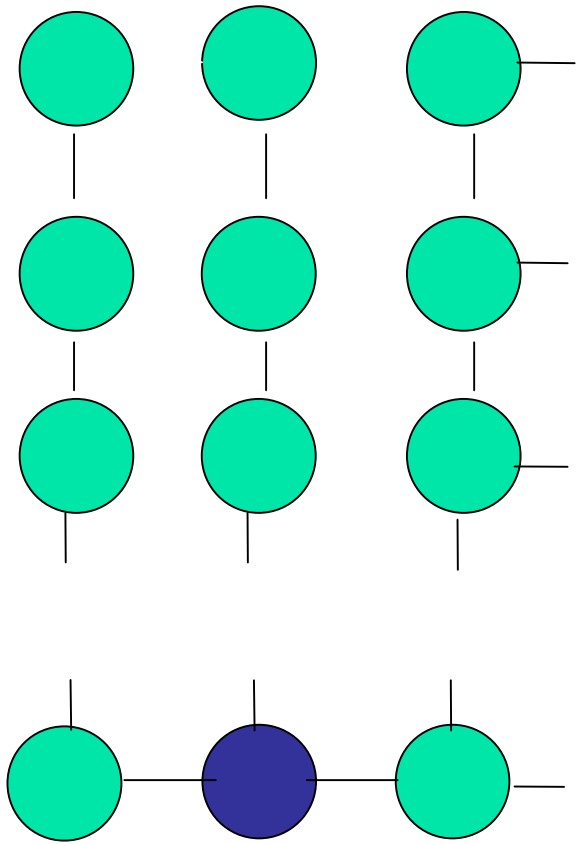
Measure after T steps

Quantum walk leads to a state in which marked S have higher amplitude

"leaky"
Quantum walk



Set of items S , some marked.



Quantum Algorithm:

Start with a uniform superposition over all S .

Apply one coin (transition rule) if S marked,
another if S not marked.

Measure after T steps

Quantum walk leads to a state in which marked S have higher amplitude

Applications :

spatial search [Ambainis, Kempe, Rivosh'05]

Communication

- Nouveaux protocoles de transmission
- Caractérisation du pouvoir des modèles de communication

Algorithmes

- Analyse des algorithmes pour isomorphisme de graphe et recherche locale

Pouvoir des machines quantiques

- Étude de la décohérence (bruit)
- Étude des modèles physiques sur réseau
- Analyse des nouveaux modèles de calcul (calcul adiabatique sur réseau)

...