

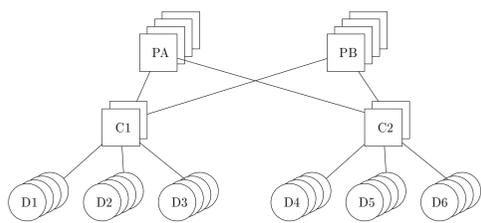
Bounding transient and steady-state dependability measures through algorithmic stochastic comparison

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Highly available multicomponent systems

- Huge state space ($> 10^{10}$ states) → excludes the generation of the whole state space
 - Most of the probability mass located in a small portion of the state space
- *bounding techniques*.



Repairable system studied by Muntz et al. 1989, Carrasco 1999.

State space is divided into UP (system is operational) and DOWN states.

Figure 1.

Goal: to compute bounds for both **transient** (new) and **steady-state** dependability measures such as reliability, point and steady-state availability.

Uniformization → *discrete time Markov chain (DTMC) bounding problem*.

Method: algorithmic construction of a numerically tractable model that can be stochastically compared with the original one.

Avoiding generation of the whole state space

To apply LMSUB algorithm: the transition matrix must be stored on disk (may be difficult).

If impossible → **LL (Lumpable and Larger) algorithm**

- Using a high level formalism designs a lumpable matrix A such that $P \preceq_{st} A$.
- Generates and stores the lumped version (B) of A (B not necessarily \preceq_{st} -monotone) → input for LMSUB.
- May depend on the high level formalism used for model specifications.

The bound obtained by LMSUB on B is also a bound of the original matrix we are not able to store.

Numerical example

System in Figure 1 with model description and parameter values as in Carrasco 1999:

- Failures and reparations of components are exponentially distributed.
- Only one processor of each type is active and only the active processor can fail. A failure of PA may contaminate the active PB.
- Two failure modes (soft and hard) and one repairman.
- UP states: at least one processor unit (PA or PB), one controller of each set (C1 and C2) and three disks of each cluster (D1 and ... and D6) are operational.

Only 36 components of 10 types yet the state-space is of order of $9.0 \cdot 10^{10}$.

Algorithmic stochastic comparison approach

Strong stochastic order (\preceq_{st}): X and Y two random variables.

$$X \preceq_{st} Y \text{ if } E[f(X)] \leq E[f(Y)], \forall f \text{ non decreasing function.}$$

Comparison of two DTMC's X and Y (with transition matrices P and Q): If $X(0) \preceq_{st} Y(0)$, $P \preceq_{st} Q$ (i.e. $P_{i,*} \preceq_{st} Q_{i,*} \forall i$) and P or $Q \preceq_{st}$ -monotone, then

$$X(k) \preceq_{st} Y(k), \forall k \text{ (if } X \text{ and } Y \text{ ergodic, } \pi_X \preceq_{st} \pi_Y).$$

Can be used to compare increasing rewards of two DTMC's.

Additionally, Q will be forced to be ordinary lumpable → *state-space size reduction*.

Construction of an \preceq_{st} monotone lumpable upper bounding chain for a finite DTMC X (with transition matrix P)

$$\sum_{k=j}^n P_{i,k} \leq \sum_{k=j}^n Q_{i,k}, \forall i, j \text{ (transition matrix comparison)}$$

$$\sum_{k=j}^n Q_{i-1,k} \leq \sum_{k=j}^n Q_{i,k}, \forall j, \forall i \geq 2 \text{ (monotonicity)}$$

For a given partition $C_l, l \in L$ of the state space, $\forall i \in L, \forall a, b \in C_i$,

$$\sum_{k \in C_j} Q_{a,k} = \sum_{k \in C_j} Q_{b,k}, \forall j \in L \text{ (lumpability)}$$

LMSUB algorithm, Fourneau et al. 2003: additional irreducibility constraints → steady-state bounds.

Bounding transient rewards

LMSUB (Lumpable Monotone Stochastic Upper Bound) algorithm

- Inspired by transient analysis: no irreducibility constraints → can be used to compute bounds even of reducible matrices.
- Simpler: based only on transition matrix comparison, monotonicity and lumpability.
- Can be used also for steady-state bounds (the bounding chain has only one recurrent class).

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for b = m downto 1 do
  for j = last(b) downto first(b) do
    for i = 1 to n do
      sumqi,j = max(sumqi-1,j, ∑k=jn Pi,k);
      if b < m and j = last(b) and i = 1 then
        sumqi,j = max(rblock(i),b+1, sumqi,j);
      end
    end
  end
  for a = 1 to m do ra,b = sumqlast(a),first(b);
end
    
```

- P , Q and R - the initial, lumpable and lumped matrix.
- Partition into m macro states.
- $first(l)$ and $last(l)$ - the first and the last state of a macro state.
- $sumq_{i,j} = \sum_{k=j}^n q_{i,k}$ with $sumq_{-1,j} = 0$.

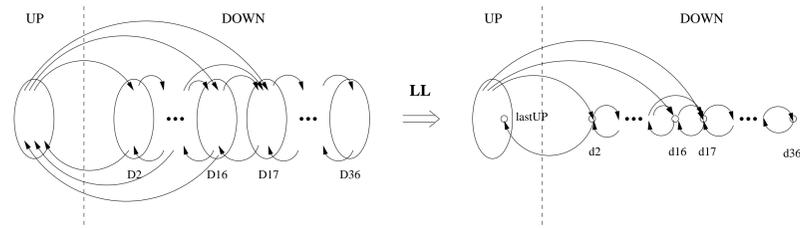
Implementation:

- Sparse representation of P and Q .
- Only R is actually computed and stored.
- Only three vectors in memory.

Availability bounds

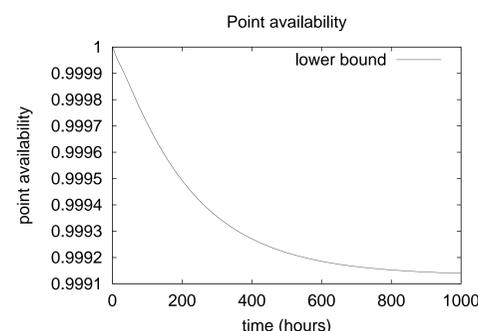
LL step:

DOWN states with the same total number of faults are aggregated (all UP states are generated - concentration of the probability mass) ⇒ 1 312 235 states.



LMSUB step:

UP states (except $lastUP$) are ordered (increasingly) and regrouped by number of failed components followed by number of hard failures; followed by $lastUP$ state, then by 35 aggregated DOWN states. ⇒ 172 subsets.

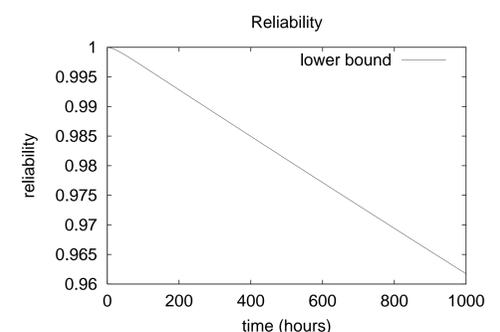


Steady-state availability lower bound: 0.999132158.

Reliability bounds

Analysis of chain with all the DOWN states aggregated into one absorbing state.

LMSUB step: Aggregation by number of failed components followed by number of hard failures → 137 blocks (1 DOWN).



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